International Capital Markets: MSc

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Economics of Capital Flows

- Consumption Smoothing
- Efficient capital allocation
- International risk sharing

- Consider the two-period SOE (small open economy) model, exogenous world interest rate r.
- Let the discount factor be $\beta = 1/(1+\delta)$.
- The household receives endowments Y_1 and Y_2 in periods 1 and 2.
- Its maximisation problem is

$$\underset{C_1}{Max}\, U = U(C_1) + \beta U(C_2)$$

subject to the present-value budget constraint

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

- This constraint can be substituted into the maximisation expression

$$\underset{C_1}{Max}\, U = U(C_1) + \beta U[(1+r)(Y_1 - C_1) + Y_2]$$

- By perturbation, the optimality condition linking C_1 and C_2 is

$$U^{'}(C_1) = \beta(1+r)U^{'}(C_2)$$

- $\beta(1+r) = 1, \qquad C_1 = C_2$
- If $\beta(1+r) > 1$, $C_2 > C_1$.
- If $\beta(1+r) < 1$, $C_2 < C_1$.

- We focus on the benchmark case of $\beta(1+r) = 1$.

$$C_2 = C_1 = \frac{1+r}{2+r}\left[Y_1 + \frac{Y_2}{1+r}\right]$$

- The current account surplus in period 1 is

$$CA_1 = Y_1 - C_1$$

- If $Y_1 = Y_2$, the current account will be zero.
- If $Y_1 < Y_2$, $CA_1 = Y_1 - C_1 < 0$.
- If $Y_1 > Y_2$, $CA_1 = Y_1 - C_1 > 0$.

Temporary versus Permanent Shocks

- Suppose $Y_1 = Y_2 = Y^0$. In this case, we expect no current account action.
- Now imagine $Y_1 = Y_2 = Y^H > Y^0$. Since the output profile remains flat $(Y_1 = Y_2)$, we still expect no current account imbalance.
- Consider $Y_1 = Y^H > Y^0 = Y_2$. Since $Y_1 > Y_2$ in this case, $CA_1 = Y_1 - C_1 > 0$.
- We expect current account imbalances in response to *temporary* shocks, but not in relation to *permanent* shocks.

Adding Government Consumption

- The government sector may operate through two channels: (a) it may alter the shape of the utility function; and (b) it reduces the resources available for private consumption.
- Let the maximisation function now be given by

$$U(C_1) + V(G_1) + \beta U(C_2) + \beta V(G_2)$$

$U^{'}(C_1) = \beta(1 + r)U^{'}(C_2)$. (Indeed, many textbook writers simplify further and just assume that government spending does not provide any private utility $V(G) = 0$.)

- In this case, government consumption only matters through its impact on the budget constraint

$$\begin{aligned} C_1 + \frac{C_2}{1+r} &= (Y_1 - G_1) + \frac{(Y_2 - G_2)}{1+r} \\ &= Y_1^N + \frac{Y_2^N}{1+r} \end{aligned}$$

- If we continue to assume that $\beta(1+r)=1$ such that $C_1 = C_2$, the optimal level of consumption is now

$$
\begin{aligned}
C_2 &= C_1 = \frac{1+r}{2+r}\left[Y_1^N + \frac{Y_2^N}{1+r}\right] \\
&= \frac{1+r}{2+r}\left[Y_1 - G_1 + \frac{Y_2 - G_2}{1+r}\right]
\end{aligned}
$$

The current account in period 1 is

$$CA_1 = (Y_1 - G_1) - C_1$$

Let us assume that output is perfectly flat $Y_1 = Y_2$.

- If $G_1 = G_2$, the current account will be zero.
- If $G_1 > G_2$, $CA_1 = (Y_1 - G_1) - C_1 < 0$.
- If $G_1 < G_2$, $CA_1 = (Y_1 - G_1) - C_1 > 0$.

Adding Capital

Now, we assume that production requires capital

$$Y = AF(K)$$

We assume that domestic capital accumulates according to

$$K_{t+1} - K_t = I_t$$

The accumulation of net foreign assets is

$$\begin{aligned} B_{t+1} - B_t &= CA_t = r_t B_t + Y_t - (C_t + G_t + I_t) \\ CA_t &= S_t - I_t \end{aligned}$$

We first consider a two-period SOE model. The current account in period 1 is just given by

$$B_2 = Y_1 - (C_1 + G_1 + I_1)$$

This implies that total spending in period 2 is

$$C_2 + G_2 + I_2 = Y_2 + (1 + r)B_2$$

These two equations can be combined to give the present-value budget constraint

$$C_1 + \frac{C_2}{(1+r)} = (Y_1 - G_1 - I_1) + \frac{(Y_2 - G_2 - I_2)}{(1+r)}$$

We can insert the budget constraint into the 2-period maximisation problem to give

$$\begin{aligned} \underset{C_1, I_1}{Max} U &= U(C_1) + \beta U[(1+r)[AF(K_1) - C_1 - G_1 - I_1] \\ &+ AF(K_1 + I_1) - G_2 + I_1 + K_1] \end{aligned}$$

The optimality condition for consumption is unchanged

$$U^{'}(C_1) = \beta(1+r)U^{'}(C_2)$$

while arbitrage between domestic capital and the foreign asset means that each should offer the same marginal return

$$AF^{'}(K_2) = r$$

- “Separation" theorem
- Suppose $K_2^* > K_1$. Such a country has two reasons to run a current account deficit: (a) $K_2^* > K_1$ implies that high investment is required; and (b) $K_2 > K_1 \Rightarrow Y_2 > Y_1$, providing an additional consumption-smoothing reason to borrow.

Global Interdependence

Global output equals global consumption

$$Y_t + Y_t^* = C_t + C_t^*$$

which means that the sum of home and foreign trade surpluses is zero

$$(Y_t - C_t) = -(Y_t^* - C_t^*)$$

Taking into account capital accumulation,

$$C_t + C_t^* + G_t + G_t^* + I_t + I_t^* = Y_t + Y_t^*$$

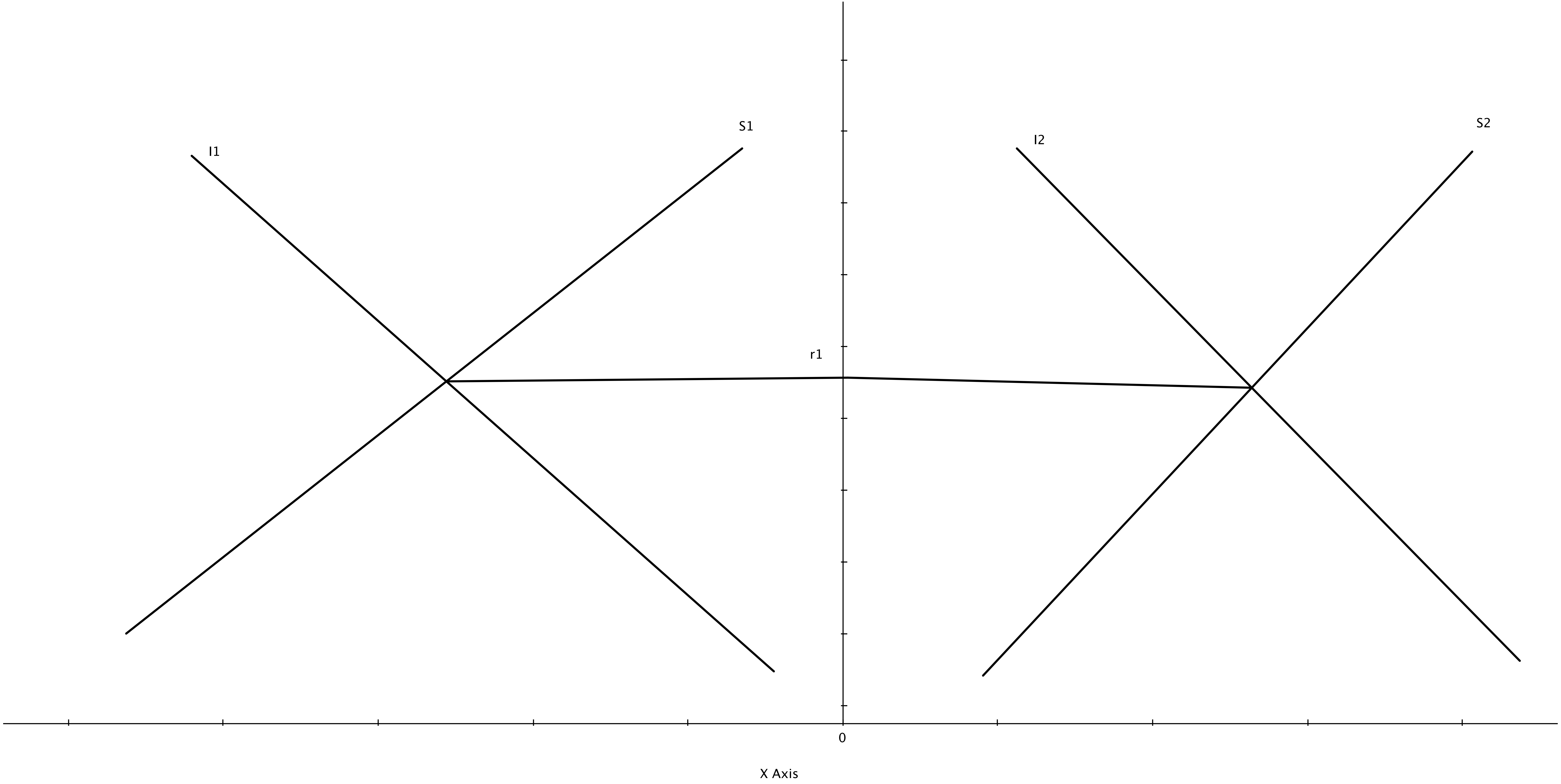
such that

$$(Y_t - C_t - G_t) - I_t = -\left[(Y_t^* - C_t^* - G_t^*) - I_t^*\right]$$

or

$$S_t - I_t = -\left[S_t^* - I_t^*\right]$$

- Global shocks: adjustment in world interest rate
- Country-specific shocks: current account adjustment
- Interdependence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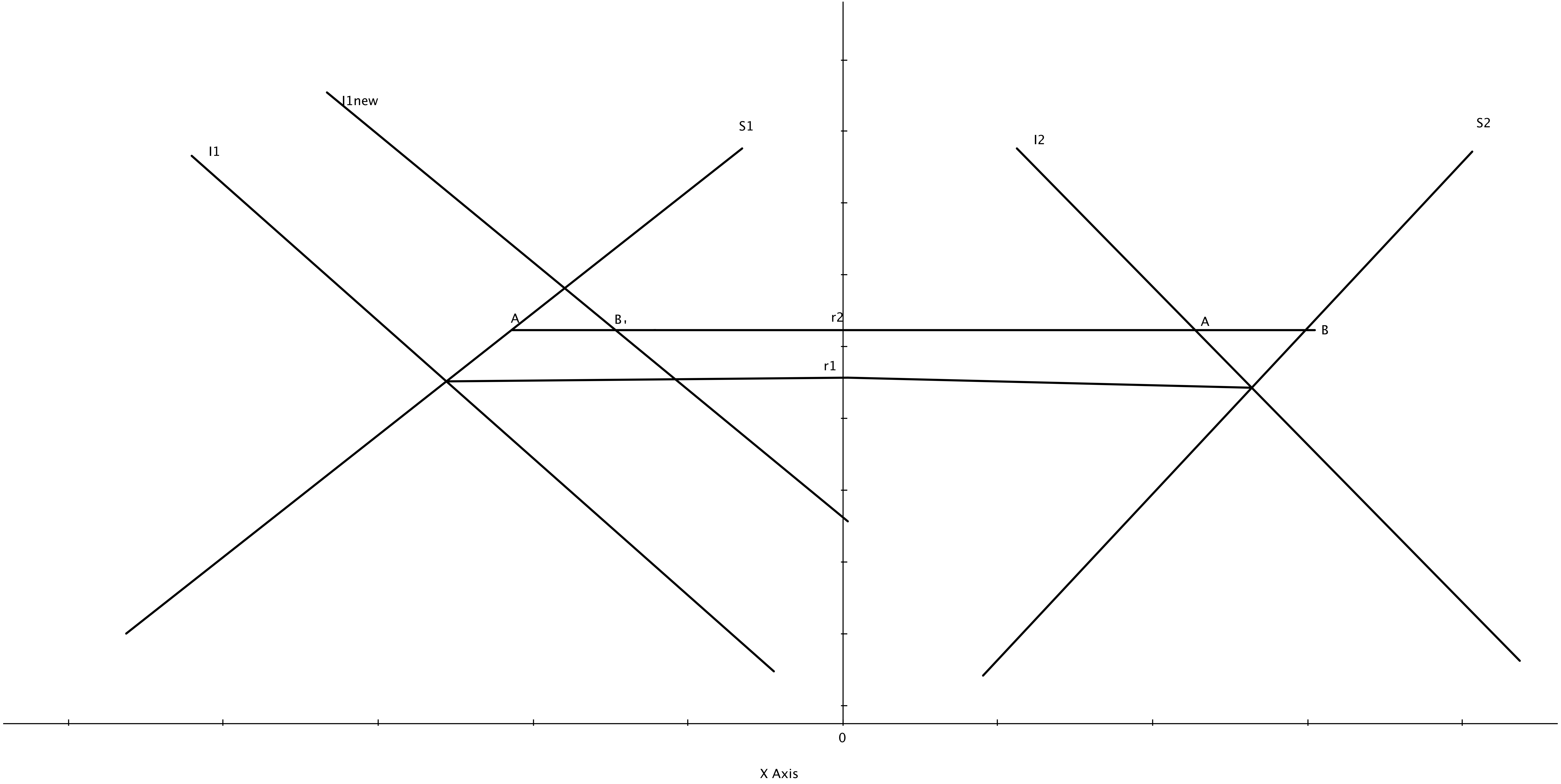
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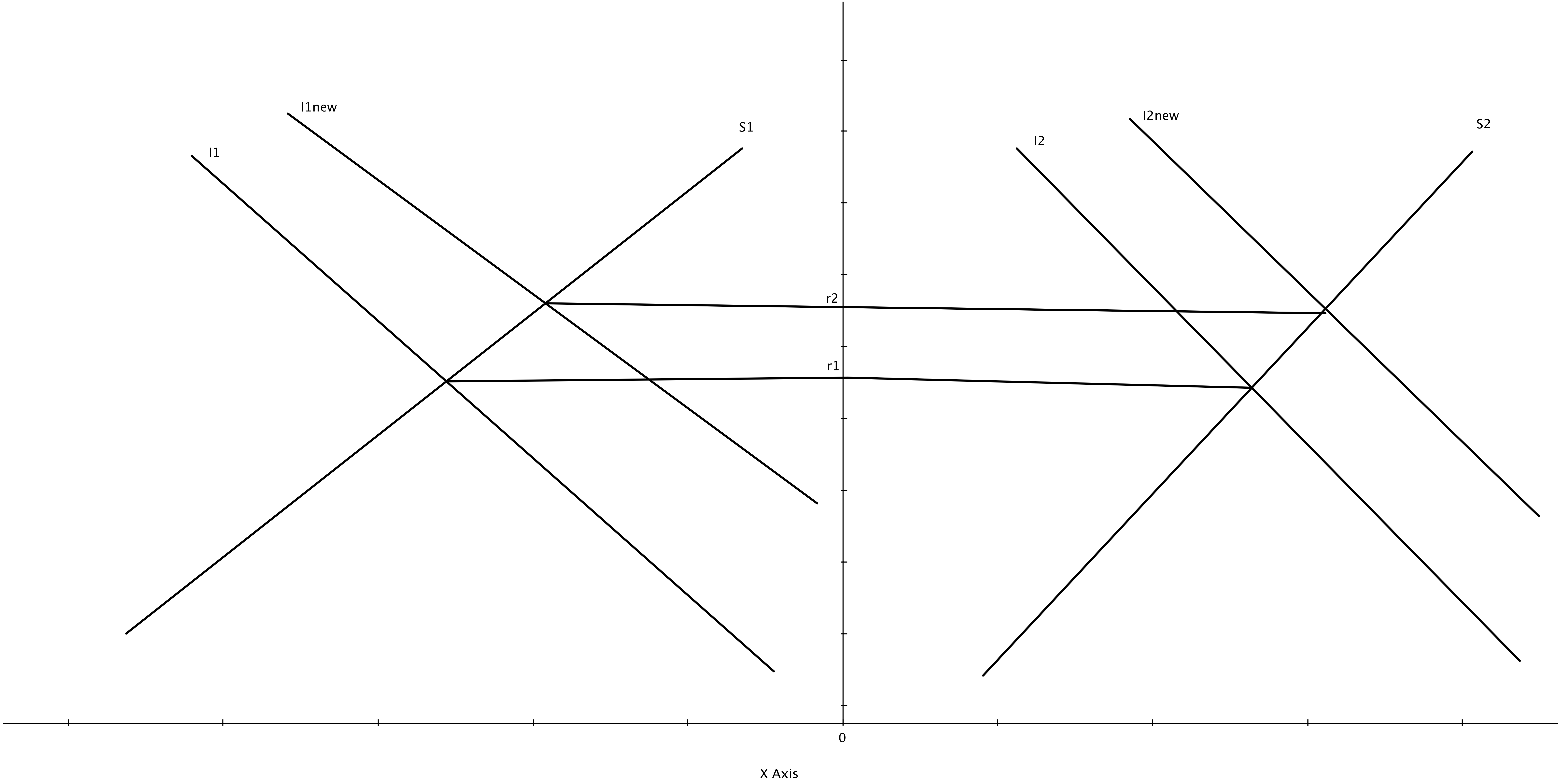
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Infinite-Horizon Model

The maximisation of utility involves

$$U_t = \sum_{s=t}^{s=\infty} \beta^{s-t} U\left[(1+r)B_s - B_{s+1} + A_s F(K_s) - (K_{s+1} - K_s) - G_s\right]$$

Optimality involves respecting the transversality condition

$$\lim_{T \to \infty} \left(\frac{1}{1+r}\right)^T B_{t+T+1} = 0$$

The first-order conditions are

$$\begin{aligned} U^{'}(C_s) &= \beta(1+r)U^{'}(C_{s+1}) \\ A_{s+1}F^{'}(K_{s+1}) &= r \end{aligned}$$

To work out the optimal level of C_t, we need to examine the present-value budget constraint

$$\sum_{s=t}^{s=\infty}\left(\frac{1}{1+r}\right)^{s-t}(C_s)=(1+r)B_t+\sum_{s=t}^{s=\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s-G_s-I_s)$$

Use the rule that the present value of a constant stream is

$$\sum_{s=t}^{s=\infty}\left\{\left(\frac{1}{1+r}\right)^{s-t}Z\right\} = Z\left\{1+\frac{1}{1+r}+\left(\frac{1}{1+r}\right)^{2}+...\right\}$$
$$= Z\left\{\frac{1}{1-\left(\frac{1}{1+r}\right)}\right\}=Z\left\{\frac{1+r}{r}\right\}$$

This allows us to write the budget constraint as

$$C_t \frac{1+r}{r} = (1+r)B_t + \sum_{s=t}^{s=\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s)$$

or

$$C_t = rB_t + \left(\frac{r}{1+r}\right) \left\{ \sum_{s=t}^{s=\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s - I_s) \right\}$$

Let us call the 'permanent' value of some variable X_t by $\tilde{X}_t$ where this is defined by

$$\sum_{s=t}^{s=\infty} \left(\frac{1}{1+r}\right)^{s-t} \tilde{X}_t = \sum_{s=t}^{s=\infty} \left(\frac{1}{1+r}\right)^{s-t} X_s$$

That is, receiving the constant value $\tilde{X}_t$ each period has the same present value as the actual series of X_s.

This means that we can write the level of consumption as

$$C_t = rB_t + \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t$$

such that consumption equals the interest income on the existing stock of net foreign assets, plus the 'permanent' ('average') value of net output. In turn, this means that the current account can be written as

$$CA_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t)$$

- "Fundamental" current account equation

Sustainability of the Net Foreign Asset Position

Another way to write the present-value budget constraint is

$$
\begin{aligned}
B_t &= \left(\frac{1}{1+r}\right)\sum_{s=t}^{s=\infty}\left(\frac{1}{1+r}\right)^{s-t}(G_s + I_s + C_s - Y_s) \\
&= -\left(\frac{1}{1+r}\right)\sum_{s=t}^{s=\infty}\left(\frac{1}{1+r}\right)^{s-t}(TB_s)
\end{aligned}
$$

- PV of trade deficits equals current NFA position
- country risk analysis

Sustainability Analysis I

Assume that GDP grows at a constant rate g

$$Y_{s+1} = (1+g)Y_s$$

For a constant B/Y ratio

$$\begin{aligned} B_{s+1} &= (1+g)B_s \\ B_{s+1} - B_s &= gB_s \end{aligned}$$

For zero valuation effects, the change in the stock of net foreign assets is given by the current account, so that

$$B_{s+1} - B_s = rB_s + TB_s = gB_s$$
$$TB_s = -(r-g)B_s$$

It follows that:

- If $r > g$, then a debtor country ($B_s < 0$) must run a trade surplus equal to $-(r-g)B_s$ in order to stabilise the ratio of external debt to GDP
- If $r < g$, then a debtor country ($B_s < 0$) can run a trade deficit equal to $-(r-g)B_s$ while maintaining a constant ratio of external debt to GDP

Sustainability Analysis II

Allow for heterogeneity in returns on assets and liabilities

$$TB_s = -\left[(r^L - g)B_s + (r^A - r^L)A_s\right]$$

Positive return premium allows a larger *TB* deficit for a given *NFA* position.

Equity-type liabilities

$$\frac{\partial r^L}{\partial g} > 0$$

- Cannot grow out of a net liability position

Habit Formation and the Current Account

- Induces persistence in current account behaviour
- Doyle (JME); Fagan and Gaspar (ECB, 2007)
- Preferences

$$U(C_s - \gamma C_{s-1})$$
$$U((1-\gamma)C_s + \gamma \Delta C_s)$$

- Current Account ($\gamma = 0$)

$$CA_t = -\sum \left(\frac{1}{1+r}\right)^{s-t} E_t \Delta NO_s$$

where $NO = Y - I - G$

Current Account ($\gamma > 0$)

$$CA_t = \gamma CA_{t-1} + \frac{\gamma}{1+r}\Delta NO_t - \left(1 - \frac{\gamma}{1+r}\right)\sum\left(\frac{1}{1+r}\right)^{s-t} E_t \Delta NO_s$$

International Risk Sharing

- A key role for international capital markets is to allow agents to diversify wealth by selling claims to domestic output and acquiring claims on foreign output.
- This is done through financial portfolios (equities, bonds, derivatives)
- the international investments of firms (FDI) and banks (international private ‘other’ debt assets and liabilities)
- international trade in commercial and residential properties (another part of FDI)
- the international positions of governments (official reserves on the asset side, external public debt on the liability side).

SOE Model

- The first-period endowment is a known value Y_1.
- The second-period endowment is uncertain: in state 1, it takes the value $Y_2(1)$ and in state 2 it takes the value $Y_2(2)$.
- The probability of state 1 is denoted by $\pi(1)$ and the probability of state 2 is denoted by $\pi(2)$ where $\pi(1) + \pi(2) = 1$.

- In this environment, households maximise the present value of expected utility from consumption

$$
\begin{aligned}
MaxU_1 &= U(C_1) + \beta EU(C_2) \\
&= U(C_1) + \beta\pi(1)U[C_2(1)] + \beta\pi(2)U[C_2(2)]
\end{aligned}
$$

We assume that a complete set of assets with state-contingent returns are traded.

- We assume that for every state s, there exists an Arrow-Debreu (AD) security that pays 1 unit if state s occurs and 0 otherwise.
- In our case, there are only two possible states in period 2 and so we only need 2 AD securities.
- Let $B_2(1)$ pay 1 unit if $s = 1$ and 0 otherwise, while $B_2(2)$ pays 1 unit if $s = 2$ and 0 otherwise.
- A household can hold a positive ('long') quantity of an AD security (receiving a payoff in that state) or a negative ('short') quantity (making a payoff in that state).

- Let the price of each AD security be written as

$$Q(s) = \frac{p(s)}{1+r}$$

In this way, the security price is composed into two parts: the numerator reflects a value that varies across states, while the numerator is common across all states.

- (This is a useful decomposition, since it is possible to obtain a risk-free return on a 'composite' portfolio by buying $(1+r)$ units of each AD security - this ensures that the investor receives $(1+r)$ in each possible state s.

- In the two-state example, the price of this 'composite' portfolio is $(1 + r) * [Q(1) + Q(2)] = [p(1) + p(2)]$.
- Imagine a riskless bond existed that paid an interest rate r. If this riskless bond existed, the cost of achieving $1 + r$ units of consumption in period 2 is 1 unit of period-1 output.
- By arbitrage, this implies that $[p(1) + p(2)] = 1$, which is a useful normalisation of the relative prices of AD securities.)

The period–1 budget constraint facing the household is

$$Y_1 - C_1 = \frac{p(1)}{1+r}B_2(1) + \frac{p(2)}{1+r}B_2(2)$$

That is, savings can be allocated to the accumulation of type-1 and type-2 AD securities.

In period 2, consumption is given by

$$C_2(s) = Y_2(s) + B_2(s)$$

Accordingly, we can write the present-value budget constraint as

$$C_1 + \frac{p(1)C_2(1) + p(2)C_2(2)}{1+r} = Y_1 + \frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}$$

where the household must decide the allocation of consumption between C_1, $C_2(1)$ and $C_2(2)$.

- Note that it always possible to eliminate volatility in period-2 consumption - that is, set $C_2(1) = C_2(2)$.
- For instance, if $C_1 = Y_1$, the present-value budget constraint tells us that $C_2(1) = C_2(2)$ can be achieved at the level $C_2(1) = C_2(2) = p(1)Y_2(1) + p(2)Y_2(2)$.

However, the economic question is whether it is *optimal* to eliminate period-2 consumption risk.

We now turn to the optimality conditions for the utility-maximisation problem. By perturbation, an optimal allocation requires that

$$U^{'}(C_1) = \pi(s)\frac{1+r}{p(s)}\beta U^{'}[C_2(s)]$$

Since there is such a condition for each state s, this also provides links across the levels of period-2 consumption $C_2(s)$ and $C_2(s^{'})$.
In the two-state case, we can sum across the two optimality conditions to get

$$\begin{aligned}[p(1)+p(2)]U^{'}(C_1) &= (1+r)\beta\left[\pi(1)U^{'}[C_2(1)]+\pi(2)U^{'}[C_2(2)]\right] \\ U^{'}(C_1) &= (1+r)\beta EU'(C_2)\end{aligned}$$

If we divide across the two optimality conditions, we get

$$\frac{\pi(1)U^{'}[C_2(1)]}{\pi(2)U'[C_2(2)]} = \frac{p(1)}{p(2)}$$

This condition tells us that it is optimal to eliminate uncertainty about period-2 consumption $[C_2(1) = C_2(2)]$ only if

$$\frac{p(1)}{p(2)} = \frac{\pi(1)}{\pi(2)}$$

That is, if state-contingent asset prices are 'actuarially fair.'

Global Model

Consider a two-country world.
The global resource constraints are

$$\begin{aligned} C_1 + C_1^* &= Y_1 + Y_1^* = Y_1^W \\ C_2(s) + C_2^*(s) &= Y_2(s) + Y_2^*(s) = Y_2^W(s) \end{aligned}$$

Home and foreign households face the same global asset prices. Let us assume that utility is of the CRRA (constant relative risk aversion) form

$$U(C) = \frac{C^{1-\rho}}{1-\rho}$$

such that $U^{'}(C) = C^{-\rho}$ and $\rho > 0$ denotes the degree of risk aversion.

Optimal consumption allocations at home and overseas involve

$$C_2(s) = C_1 \left[\beta\pi(s)\frac{(1+r)}{p(s)}\right]^{1/\rho}$$
$$C_2^*(s) = C_1^* \left[\beta\pi(s)\frac{(1+r)}{p(s)}\right]^{1/\rho}$$

We can sum across these conditions to get

$$Y_2^W(s) = Y_1^W \left[\beta\pi(s)\frac{(1+r)}{p(s)}\right]^{1/\rho}$$

This can be re-arranged as

$$\frac{p(s)}{1+r} = \beta\pi(s)\left[\frac{Y_2^W(s)}{Y_1^W}\right]^{-\rho}$$

so that relative asset prices across states s and $s^{'}$ are

$$\frac{p(s)}{p(s^{'})} = \frac{\pi(s)}{\pi(s^{'})}\left[\frac{Y_2^W(s)}{Y_2^W(s^{'})}\right]^{-\rho}$$

We can see that asset prices are only 'actuarially fair' if

$$Y_2^W(s) = Y_2^W(s^{'})$$

That is, if the period-2 endowment is risk-less from a global perspective.

It remains to determined the level of the 'common' component in asset prices $(1+r)$. First, we eliminate $(1+r)$ from the expression for the asset price for some state $s^{;}$. To do this, recognise that

$$\sum_{s=1}^{s=S} p(s) = 1$$

so that

$$\begin{aligned} p(s^{'}) &= 1 - \sum_{s \neq s^{;}} p(s) \\ &= 1 - p(s^{'}) \sum_{s \neq s^{;}} \frac{\pi(s)}{\pi(s^{;})} \left[\frac{Y_2^W(s)}{Y_2^W(s')} \right]^{-\rho} \end{aligned}$$

which can be re-arranged to give

$$p(s') = \frac{\pi(s')[Y_2^W(s')]^{-\rho}}{\sum_{s=1}^{s=S} \pi(s)[Y_2^W(s)]^{-\rho}}$$

Since the optimality condition for consumption implies that

$$\frac{p(s^{'})}{1+r} = \pi(s^{'})\beta \left[\frac{Y_2^W(s)}{Y_1^W}\right]^{-\rho}$$

We can combine these equations to get

$$(1+r) = \frac{(Y_1^W)^{-\rho}}{\beta \sum_{s=1}^{s=S} \pi(s)[Y_2^W(s)]^{-\rho}}$$

Consumption Correlations under International Risk Sharing

Since home and foreign households face the same set of globally-determined asset prices, they will make the same relative allocations:

$$\frac{C_2(s)}{C_2(s')} = \frac{C_2^*(s)}{C_2^*(s')} = \frac{Y_2^W(s)}{Y_2^W(s')}$$

where the last equality holds due to the adding-up constraint.

Similarly,

$$\frac{C_2(s)}{C_1} = \frac{C_2^*(s)}{C_1^*} = \frac{Y_2^W(s)}{Y_1^W}$$

It follows that fluctuations in home and foreign consumption are perfectly correlated, where the fluctuations just reflect shifts in the global endowment.

Each consumes a constant share of world output, with the home share given by

$$\frac{C_2(s)}{Y_2^W(s)} = \frac{C_1}{Y_1^W} = \mu$$

where

$$\mu = \left[\frac{Y_1 + \sum_{s=1}^{s=S} \frac{p(s)Y_2(s)}{(1+r)}}{Y_1^W + \sum_{s=1}^{s=S} \frac{p(s)Y_2^W(s)}{(1+r)}} \right]$$

That is, the home country commands a higher share, the greater is its share in the present value of world output.

- The prediction that home and foreign consumption growth rates are perfectly correlated is strongly rejected in the data
- Many reasons why the scale of international risk sharing is less than predicted by the 'perfect risk sharing' framework
  - Differences in consumption risk (trade frictions generate asset-market frictions)
  - Differences in 'non-tradable endowment' risk (e.g. non-diversifiable labour income risk)
  - Taxation differences
  - Differences in legal treatment and corporate governance
  - Asymmetric information
  - Behaviourial finance: familiarity bias; learning costs